ABOUT THE SPECTRAL CLASSIFICATION OF Be STARS

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RESUMEN: La Teoría de la Información permite desarrollar una herramienta útil para medir la interdependencia en la información que brindan un grupo de clasificaciones. Aplicaremos aquí el método propuesto por Estabrook (1967) a la muestra de 140 estrellas Be del trabajo de Jaschek et al. (1980), estudiando las clasificaciones por tipo espectral, en relación al sistema clasificatorio de las Be propuesto por Jaschek, en el trabajo citado.

ABSTRACT: The Information Theory is a useful development tool to measure the correlation in the information of several classifications over the same of objects. In this work, we apply the method proposed by Estabrook (1967) to the sample of 140 Be stars subdivided in five groups by Jaschek et al. (1980). We study the possible dependence between this classification of Be stars (J) and the MK spectral classification (T).

1. FUNDAMENTAL CONCEPTS

Table I gives the numbers of those Be stars which share the same classification. We can associate, with each "state" of Jaschek's classification (J), a distribution of probability $P(J_i)$. That is, the probability of choosing an object at random from the collection of Be stars to which a description has been assigned. The related expression is:

$$P(J_i) = \frac{\text{Number of Be stars with } J_i \text{ classification}}{\text{Total number of Be stars}}$$

In the same way, we can obtain $P(T_m)$ applied to the selection of stars with spectral type T_m. The results are presented in Tables 2 and 3.

TABLE 1

Description of the stars which share the same classification (J and T).

T_m \ J_i	I	II	III	IV	V
B 0	3	-	-	-	-
B 0.5	2	-	-	-	1
B 1	12	-	-	-	3
B 2	17	1	-	1	8
B 3	6	5	-	4	7
B 4	2	-	1	1	2
B 5	1	-	1	1	3
B 6	1	3	-	6	4
B 7	-	3	-	7	5
B 8	-	1	3	12	2
B 9	-	-	1	5	1
B 9.5	-	-	-	1	1
A 0	-	-	1	1	-

It is possible to assign to each of these discrete distributions of probability a measure of entropy according to the Information Theory (IT). This is essentially a measure of how difficult it is to predict which one of the classifications's possible descriptions is applicable to some object randomly chosen from or set. Depending on their associated distributions of probability, different classifications will be assigned to different measures of entropy (Estabrook, 1967).

TABLE 2

Distribution of the probability associated with the classification J_i

J_i	$P(J_i)$
I	0.314
II	0.093
III	0.050
IV	0.279
V	0.264

TABLE 3

Distribution of the probability associated with the classification T_m

T_m	$P(T_m)$
B 0	0.0214
B 0.5	0.0214
B 1	0.1071
B 2	0.1929
B 3	0.1571
B 4	0.0429
B 5	0.0429
B 6	0.1000
B 7	0.1071
B 8	0.1286
B 9	0.0500
B 9.5	0.0143
A 0	0.0143

TABLE 4

Conditional probabilities associated with classification J

T_m \ J_i	I	II	III	IV	V	$H(J/T_m)$
B 0	1	—	—	—	—	0
B 0.5	0.667	—	—	—	0.333	0.636
B 1	0.800	—	—	—	0.200	0.500
B 2	0.630	0.037	—	0.037	0.296	0.896
B 3	0.273	0.273	—	0.182	0.318	0.138
B 4	0.333	—	0.167	0.167	0.333	0.133
B 5	0.167	—	0.167	0.167	0.500	1.243
B 6	0.167	0.214	—	0.429	0.286	1.350
B 7	—	0.200	—	0.467	0.333	1.044
B 8	—	0.055	0.167	0.667	0.111	0.974
B 9	—	—	0.143	0.714	0.143	0.796
B 9.5	—	—	—	0.500	0.500	0.693
A 0	—	—	0.500	0.500	—	0.693

As was pointed out above, the information learned by assigning a classification to our set is equal to its initial entropy. We are now interested in learning about the classification called J. The measure of confusion (entropy) assigned to J is an upper limit of the information that can be learned about J. If, in an attempt to learn about J, we assign the classification T, we will be only successful insofar as T contains information in common with J. After T is given, it is possible with the IT, to determine the conditional entropy in J. That is, the confusion determined by the conditional probability distributions for J results from inspecting each of the states of T. Clearly, the conditional confusion in J is, either equal to the original confusion in J, or less than the original confusion in J. In the former case J and T share no common information, but in the latter one, we derive a measure of the information held in common by J and T as the difference of the original confusion in J minus the conditional confusion in J.

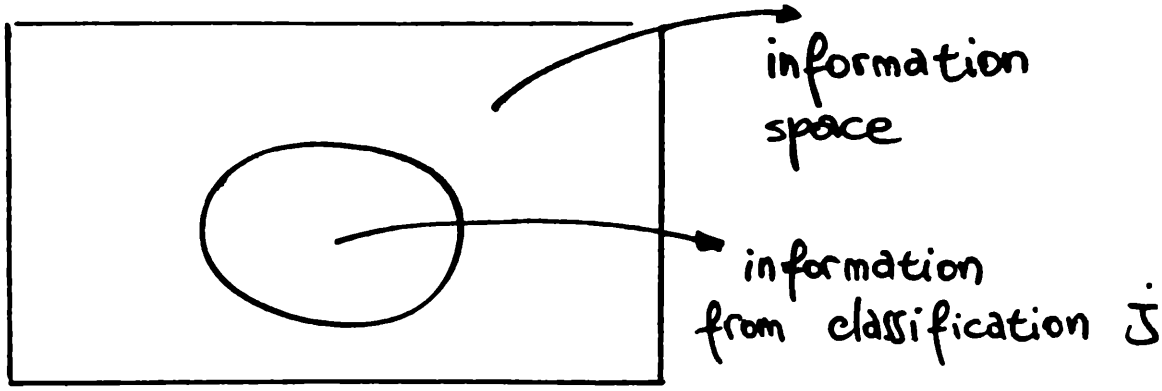
It is now possible to compute a measure of the interdependence from confusion measures with the following:

$$d(J,T) = \frac{\text{Infor. held exclusively by J + Infor. held exclusively by T}}{\text{Total information possessed by both J and T}}$$

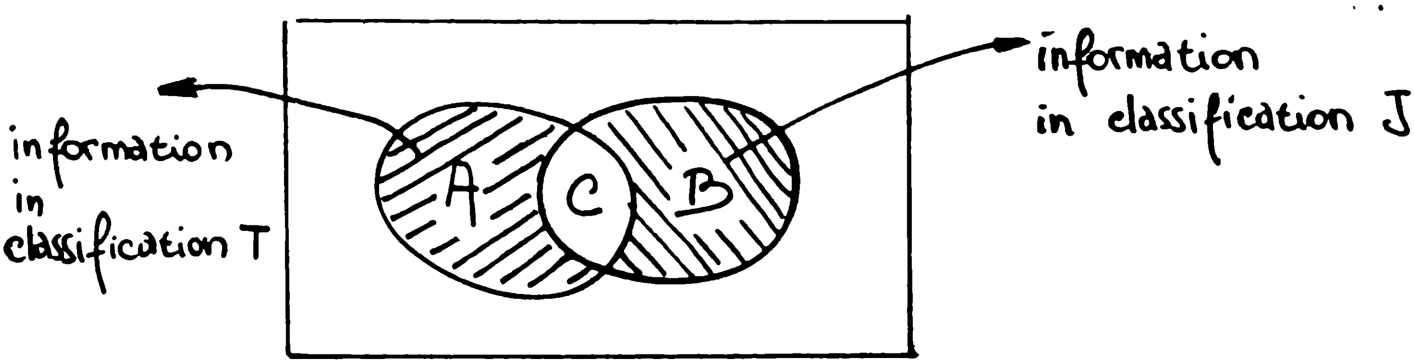
If its result is zero, J and T are identical. On the contrary, if its value is one, J and T are completely independent.

2. A SCHEMATIC VISUALIZATION OF INTERDEPENDENT MEASURE

In order to provide an intuitive understanding of the interdependent measures, the purely heuristic concept of an "Information space" will be used (Estabrook, 1967). Let us represent all the information about a study contained in the classification which is described as the area of a rectangle. Then, the information included in any single classification, J, can be represented as a subset of this rectangle (Figure 1).



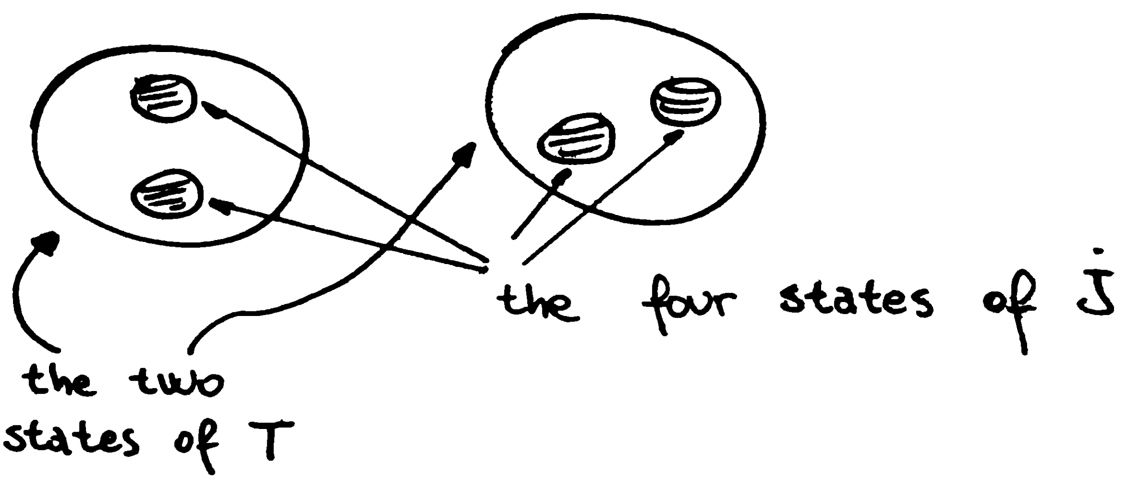
The classification J and T can be represented on the information space as two sets (Figure 2). The information held in common by the classifications J and T is labelled C. Then, A represents the information held exclusively by T, and B that information held exclusively by J.



The measure of interdependence d(J,T) can be abstraclty expressed as:

$$d(J,T) = \frac{A + B}{A+B+C}$$

If A and B are both empty (or nearly so), the classifications J and T are the same. In other words J and T are highly correlated. If A is empty or nearly so, but B is not, this means that the information contained in J includes the information contained in T. In this case J is a refinement of T. That means that the classification represented by J contains the classification represented by T (Figure 3).



2. APPLICATION

Jaschek's classification: J_i, with i = I, II, III, IV and V.

MK spectral type classification: T_m, with m = B0, B0.5, B1, B2, B3, B4, B5, B6, B7, B8, B9, B9.5, and A0.

Table 1: Description of the stars which share the same classification (J and T).

Table 2: Distribution of the probability associated with the classification J_i.

The unconditional entropy of J_i is:

$$H(J_i) = -\sum_i P(J_i) \cdot Ln(P(J_i)) = 1.442 \text{ nat}$$

Here, Ln denotes the natural logaritm. Then, the unit of the entropy is nats, (Abramson, 1981).

Table 3: Distribution of the probability associated with the classification T_m. The unconditional entropy of T_m is:

$$H(T_m) = -\sum_m P(T_m).Ln(P(T_m)) = 3.768 \text{ nat}$$

Table 4: Conditional probabilities associated with classification J.

In this table, the latest column denotes the entropy of those classifications, calculated as follows:

$$H(J/T_m) = -\sum_i P(J_i/T_m).Ln(P(J_i/T_m))$$

4. CALCULATION OF THE CONDITIONAL ENTROPY AND THE MEASURE OF THE INTERDEPENDENCE

The conditional entropy of classification J given classification T is:

$$H(J/T) = \sum_m H(J/T_m).P(T_m) = 0.752 \text{ nat}$$

Then,

from the 1.442 units of information in classification J → 0.752 belongs exclusively to classification J; → 0.690 are shared with classification T

While,

from the 3.768 units of information in classification T → 3.078 belongs exclusively to classification T; → 0.690 are shared with classification J

Therefore,

$$d(J,T) = \frac{0.752 + 3.078}{0.752+3.078+0.690}$$

$$\boxed{d(J,T) = 0.883}$$

CONCLUSIONS

From this analysis results:

1) Both classifications, J and T, share in a small portion of the information about the characteristics of Be stars.
2) As the value d(J,T) = 0.883, both classifications are highly independent. This means that to derive one group or state from one of the classifications, knowing the group or state in the other classification, has not meaning in terms of IT.
3) The exclusive information of Jaschek's classification is greater than the exclusive information of MK spectral classification, according to the results obtained from the entropy.

Finally, according to this objetive comparison, we suggest that the Be stars must be analyzed following the scheme given by Jaschek et al., because this one presents the minimal confusion (maximal information) and permits a complete description of the characteristics of the Be stars. Moreover, the simultaneous use of both classifications could be a result of undesirable redundancy to derive some properties of these objects.

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